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November 2019

MARKET EFFICIENCY: VALUE ADDED BY LARGE CAP. AND SMALL CAP. U.S. EQUITY PORTFOLIOS

CEM Benchmarking

Table of Contents

Market efficiency: Value added by large cap. and small cap. U.S. equity portfolios

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1 Introduction

In a theoretically ideal market in which participants have equal access and understanding of information, efficiency should occur [Ref. 1]. In this environment, actively managed portfolios should, on average, not be able to outperform indexed portfolios. Thus, the value added by actively managed portfolios – the difference between their return and a properly chosen benchmark – should be zero before costs. The review paper, "The Efficient Market Hypothesis and Its Critics" [Ref. 2] by the theory's originator, Burton Malkiel, offers a comprehensive review and rebuttal of the modern critiques of the theory.

A dramatic display of market inefficiency is found in the "SPIVA Institutional Scorecard: How Much Do Fees Affect the Active Versus Passive Debate" [Ref. 3] published by S&P Dow Jones Indices (referred to in what follows as the SPIVA scorecard). Here, market inefficiency works in the opposite direction to what one might have expected: persistent *underperformance* by mutual funds and institutional accounts both gross and net of investment costs. The underperformance is measured in the SPIVA scorecard by the fraction of portfolios / accounts that underperform their benchmark. Underperformance on average here extends across small, mid, and large cap. U.S. equities, value and growth tilts, emerging and global equities and several fixed income asset classes as well with only a handful of exceptions. The persistent underperformance begs the question, "who is winning?".

At CEM benchmarking, we are in an advantageous position to add information to the debate. For one, our performance database skews towards the largest institutional investors, a known source of market inefficiency in the form of outperformance at the total fund level (in Ref. 4 we have shown that larger funds with more internal and active management outperform their benchmarks net of costs). Second, the CEM database is known to be free of survivorship bias [Ref. 5]. One reason for the lack of bias is that performance is not the primary reason clients benchmark with us, but rather to understand and compare investment costs. A second is that poor returns at a pension fund result in changes to fund management, where at an individual fund manager they may trigger redemptions and closure.

Here we present in depth results on the historical performance of actively managed portfolios for two selected asset classes – dedicated U.S. large cap. equity portfolios and dedicated U.S. small cap. equity portfolios – for a group of funds that might be expected to have advantages over other investors, namely large defined benefit (DB) pension funds in the United States.

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2 Summary of results

2.1 Large cap. U.S. equity portfolios exhibited efficient-market behavior gross of costs

- Actively managed U.S. large cap. equity portfolios of large U.S. DB pension funds between 1996 and 2017 exhibit efficient-market behavior gross of costs. The average annual gross value added over the sample period was -0.02 +/- 0.13 percent, consistent with zero. The precision is achievable due to the size of the CEM database (total assets in the sample adjusted for inflation to 2017 U.S. dollars is $2,725 billion USD).
- Likewise, nearly half (48.6 percent) of these portfolios outperformed their benchmark gross of costs, indicative of market efficiency. This is somewhat higher than the just over 30 percent of large cap. U.S. equity mutual funds and institutional accounts found to have outperformed their benchmarks gross of costs in the SPIVA scorecard [Ref. 3].

2.2 Large cap. U.S. equity portfolios underperformed the market net of costs

- After deducting costs, actively managed U.S. large cap. equity portfolios had an average net value added of -0.37 +/- 0.13 percent, slightly, but significantly below zero.
- Under half (45.8 percent) of these portfolios outperformed their benchmarks. Though below 50%, it is still much higher than the only 15 percent of mutual fund managers and 20 percent of institutional accounts found to have outperformed their benchmarks net of costs in the SPIVA scorecard [Ref. 3]. This difference between our result and the SPIVA scorecard can be attributed to differences in the two samples, as our sample is heavily tilted towards the largest U.S. institutional investors who can invest at much lower cost than mutual funds and institutional accounts.
- The sample size of 1,241 portfolios is large enough to conclude at a 99.8 percent confidence level that, net of costs, large cap. U.S. equity portfolios held by large pension funds had a less than a 50 percent chance of outperforming the market.

2.3 Small cap. U.S. equity portfolios outperformed the market gross of costs

- By contrast, between 1996 and 2017, actively managed U.S. small cap. portfolios of large U.S. DB pension funds, on average, outperformed the market gross of costs. The average gross net value added over the sample period was +1.20 +/- 0.18 percent, a precision achievable due to the size of the CEM database (total assets in the sample adjusted for inflation to 2017 U.S. dollars is $1,284 billion USD).
- Over half (55.9 percent) of these portfolios outperformed their benchmark gross of costs. Again, this is much higher than the 20 percent of small cap. U.S. equity mutual funds and institutional accounts that outperformed their benchmarks gross of costs in the SPIVA scorecard [Ref. 3].
- The sample size of 2,172 funds is large enough to conclude at a precision of five nines[2] that small cap. U.S. equity portfolios held by large pension funds had a better than 50 percent chance of outperforming the market before costs and that markets were inefficient and beatable.

2.4 Small cap U.S. equity portfolios outperformed the market net of costs

- After deducting costs, actively managed U.S. small cap. equity portfolios of funds in the CEM database had an average net value added of +0.53 +/- 0.18 percent, still outperforming the market.

[2] 99.99999 percent confidence.

- Almost exactly half (49.9 percent) of portfolios outperformed their benchmarks indicating that portfolio outperformance was essentially a 50/50 proposition. Again, this is much higher than the 4 percent of small cap. U.S. equity mutual fund managers and 9 percent of small cap. U.S. equity institutional accounts that outperformed their benchmarks net of costs in the SPIVA scorecard [Ref. 3]. This difference between our result and the SPIVA scorecard may be attributed to: (i) lower investment management costs incurred by large institutional investors comprising our sample relative to the SPIVA scorecard sample, and (ii) differences in benchmarks – 86 percent of our sample benchmarked their U.S. small cap. portfolios against the Russell 2000 during 2017 whereas the benchmark used in the SPIVA scorecard is the S&P 600, known to outperform the Russell 2000 over long periods of time [Ref. 6].
- The apparent contradiction that, on average, small cap. U.S. equity portfolios outperformed their benchmarks net of costs yet had only a 49.9 percent probability of doing so is reconciled as follows: The distribution of net value added for U.S. small cap. equity portfolios *is skewed towards positive net value added*. The skewed nature of the distribution is discussed in section 4.

3 The CEM database

We at CEM Benchmarking, headquartered in Toronto, Canada, have been benchmarking global blue-chip pension funds and other large institutional investors since 1992. The core focus at CEM Benchmarking has always been benchmarking value: investment cost relative to investment performance. Currently, over $10 trillion (USD) worth of institutional money representing 350+ separate funds participate in CEM Benchmarking's annual Investment Benchmarking Service, and well over 1,000 unique funds have participated in the service at one time or another. Discussion of the data used in this research and statistical methodology can be found in Appendix A.

4 Results

4.1 Asset class characteristics

Exhibit 1 and 2 contain statistics regarding the two asset classes studied. Under the headings 'Asset class characteristics', details are displayed such as number of years of data (22), number of unique funds included in the sample (293 for large cap., 437 for small cap.), number of unique fund / year observations (1,241 for large cap., vs. 2,172 for small cap.), and total AUM adjusted to 2017 USD for inflation ($2,725 billion USD for large cap. vs. $1,284 billion for small cap.). That the number of funds/observations for large cap. is smaller than for small cap. is due to the simple fact that many funds report their large cap. U.S. equity portfolios to CEM together with small cap. U.S. in a single category 'broad U.S. equities'. While this category is dominated by large cap. U.S. equity holdings, we have excluded them from this research due to our inability to reliably split the two.

Also displayed are the 2017 actively managed fraction (49 percent for large cap. vs. 87 percent for small cap.), internally managed fraction (51 percent for large cap. vs. 8 percent for small cap.), and the most common benchmark used to measure value added (the difference between portfolio return and benchmark return). The most common benchmark for U.S. large cap. equity portfolios is the S&P 500 whereas for U.S. small cap. equity portfolios the most common benchmark is the Russell 2000.

Exhibit 1. Large capitalization U.S. equity portfolio statistics

Summary

CEM data on large capitalization (cap.) U.S. equities for U.S. (primarily) DB pension funds spanning 1996-2017 consists of portfolio level returns, asset class benchmark returns, holdings and investment costs. The data excludes the more commonly utilized broad U.S. equities aggregate asset class which, while populated primarily by large cap. U.S. equities, contains in addition mid and small cap. equity holdings.

Asset class characteristics

of years: 22
# of unique funds reporting data: 293
# of fund / year observations: 1,241
Total asset class AUM (1996 - 2017)[1]: $2,725 billion (USD)
Actively managed fraction (2017): 49%
Internally managed fraction (2017): 51%
Common benchmarks (2017): S&P 500 (54%)
Average annualized benchmark return[2] (1996 - 2017): 9.10%

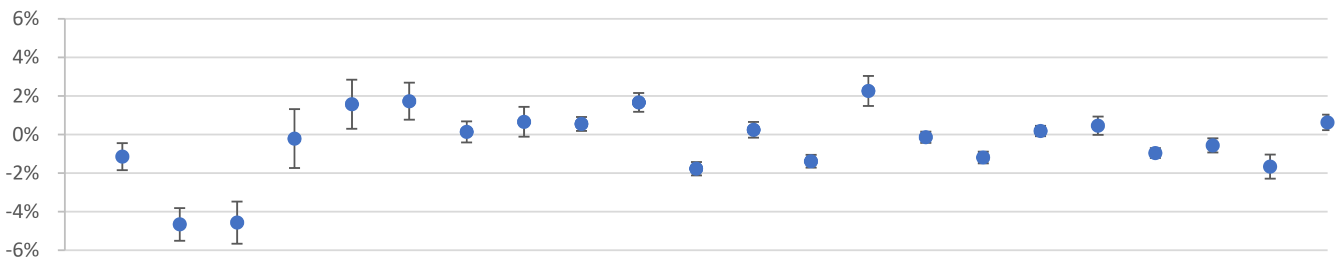
Performance summary, gross (1996 - 2017, ex. indexed)

Average annual gross value added[3]: -0.02% ± 0.13%
Proportion outperforming benchmark (POB), gross: 48.6%
Odds actual gross POB > 50%[4]: 16.0%

Performance summary, net (1996 - 2017, ex. indexed)

Average annual net value added[3]: -0.37% ± 0.13%
Proportion outperforming benchmark (POB), net: 45.8%
Odds actual net POB > 50%[4]: 0.2%

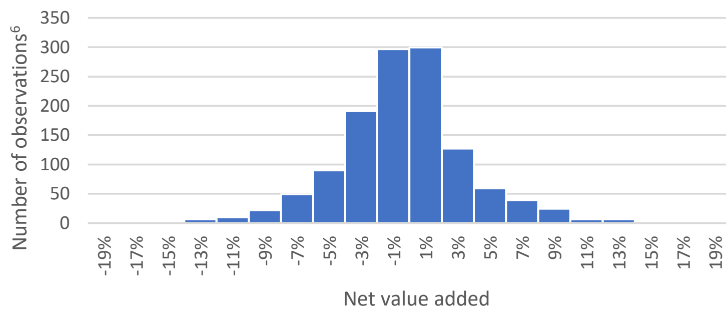
Net value added by year (fund averaged, excludes indexed holdings)



Year:	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Avg.[5] (%):	-1.15	-4.66	-4.57	-0.21	1.57	1.73	0.14	0.66	0.55	1.67	-1.77	0.25	-1.39	2.26	-0.14	-1.19	0.18	0.46	-0.96	-0.56	-1.66	0.63
Std. error[5] (%):	0.70	0.85	1.09	1.53	1.27	0.96	0.55	0.77	0.36	0.49	0.35	0.41	0.33	0.78	0.29	0.30	0.27	0.47	0.26	0.37	0.62	0.40
Std. dev.[5] (%):	2.72	3.49	5.13	7.48	6.36	4.99	2.84	4.17	2.17	3.20	2.43	4.02	3.48	7.59	2.95	3.00	2.61	4.35	2.20	2.97	4.72	2.93
Skewness[5]:	0.31	-0.94	0.30	1.09	-1.35	0.11	-0.25	-0.05	-0.48	0.04	-1.24	1.09	0.82	-2.38	1.31	-0.35	0.09	-3.30	-1.21	-2.63	4.34	-0.22
Count[5] (#):	15	17	22	24	25	27	27	29	36	43	49	97	110	95	102	97	94	84	73	65	57	53
Count > 0[5] (#):	6	2	3	8	15	17	18	15	22	32	14	47	36	62	43	29	50	56	22	34	12	33
POB >50%[4] (%):	30	0	0	8	89	94	97	64	93	100	0	42	0	100	7	0	76	100	0	69	0	97

Net value added histograms and statistics (ex. indexed)

		Percentiles[6] (%)	
Avg.[3,6]:	-0.37%	100th:	27.96%
Std. error[6]:	0.13%	90th:	4.70%
Std. dev.[6]:	4.54%	75th:	1.63%
Skewness[6]:	-0.18	50th:	-0.33%
Count (#):	1241	25th:	-2.51%
Count[6] > 0 (#):	569	10th:	-5.33%
POB >50%[4,6]:	0.2%	0th:	-41.14%



Footnotes

1. Holdings from pre-2017 are inflation adjusted to 2017.
2. Average annualized benchmark return is the compound (geometric) average return of the annual fund-averaged benchmark returns as-reported to CEM Benchmarking.
3. Average annualized gross/net value added is the equal fund-weighted gross/net value added by year (i.e., Avg. defined in footnote 5) averaged with equal weight across years. Equal weighting years (as opposed to giving equal weight to all observations) is appropriate because statistically significant, non-zero, gross/net value added is observed in several years which demonstrates that observations from different years are not drawn from the same independent identical distributed random variables.
4. Odds actual proportion outperforming benchmark > 50% (POB > 50%) is the probability that the actual (rather than observed sample) proportion of portfolios outperforming the benchmark is greater than 50%. It combines the observed POB with the number of observations. A POB >50% above ninety-five (95) percent indicates there exists a high probability that some combination of profitable skill/strategy/bias led to the observed number of portfolios outperforming their benchmark.
5. Average (Avg.) net value added is the equal fund-weighted net value added by year. Standard error (std. error) is the one-sigma standard error on the fund average average net value added, equal to the standard deviation (std. dev.) divided by the square root of the number of observation (count). Skewness (skew.) is the Pearson's moment coefficient of skewness. The count greater than zero (count > 0) is the number of observations with net value added greater than zero.
6. Data and statistics amalgamated across years have been re-weighted by the inverse of the number of observations in a year. This is done to remove sampling biases causes by variations in the number of portfolios per year appearing in the dataset. Re-weighting does not change any conclusion we have drawn from our analysis.

Exhibit 2. Small capitalization U.S. equity portfolios statistics

Summary

CEM data on small capitalization (cap.) U.S. equities for U.S. (primarily) DB pension funds spanning 1996 - 2017 consists of portfolio level returns, asset class benchmark descriptions and returns, holdings, and investment costs.

Asset class characteristics

of years: 22
# of unique funds reporting data: 437
# of fund / year observations: 2,172
Total asset class AUM (1996 - 2017)[1]: $1,284 billion (USD)
Actively managed fraction (2017): 87%
Internally managed fraction (2017): 8%
Common benchmarks (2017): Russell 2000 (86%)
Average annualized benchmark return[2] (1996 - 2017): 9.17%

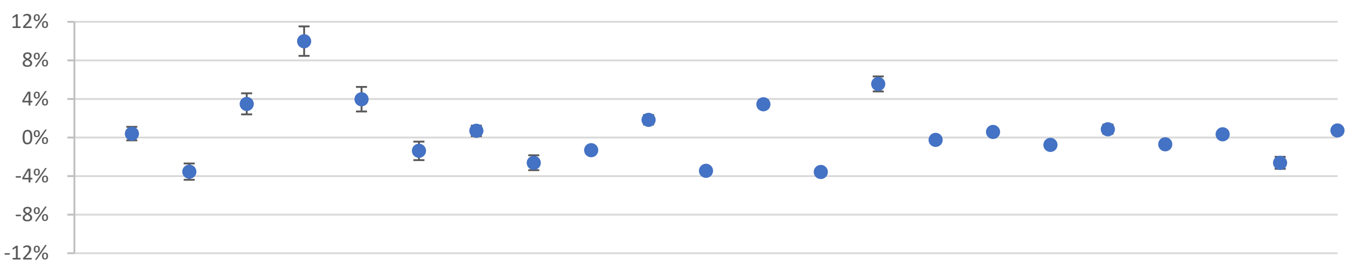
Performance summary, gross (1996 - 2017, ex. indexed)

Average annual gross value added[3]: 1.20% ± 0.18%
Proportion outperforming benchmark (POB), gross: 55.9%
Odds actual gross POB > 50%[4]: 100.0%

Performance summary, net (1996 - 2017, ex. indexed)

Average annual net value added[3]: 0.53% ± 0.18%
Proportion outperforming benchmark (POB), net: 49.9%
Odds actual net POB > 50%[4]: 46.6%

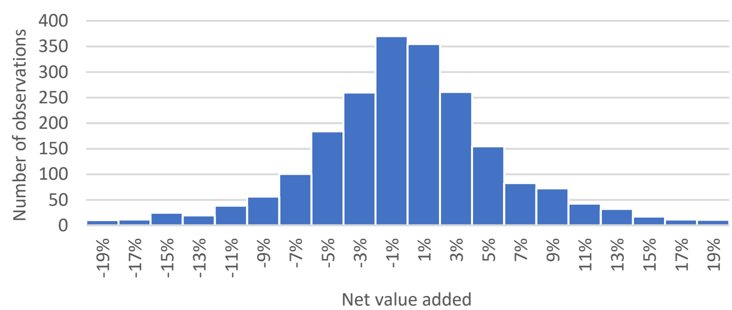
Net value added by year (fund averaged, excludes indexed holdings)



Year:	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Avg.[5] (%):	0.41	-3.54	3.50	10.00	3.98	-1.38	0.69	-2.61	-1.30	1.84	-3.46	3.45	-3.59	5.56	-0.26	0.59	-0.76	0.87	-0.72	0.33	-2.62	0.73
Std. error[5] (%):	0.70	0.97	1.05	2.47	1.22	1.13	0.57	0.83	0.42	0.32	0.25	0.51	0.37	0.73	0.36	0.28	0.25	0.41	0.28	0.34	0.62	0.45
Std. dev.[5] (%):	6.84	8.13	9.12	24.22	11.50	10.54	5.06	7.41	3.92	3.00	2.42	5.66	4.10	8.12	3.98	3.08	2.79	4.56	2.81	3.36	5.85	4.05
Skewness[5]:	0.16	-0.78	0.25	1.07	-0.57	0.77	-0.79	-0.32	-0.18	0.29	0.90	-0.21	0.56	-0.67	-0.47	0.28	0.04	-0.48	0.17	-1.60	-0.90	0.50
Count[5] (#):	95	70	76	96	89	87	80	79	89	89	96	125	124	122	125	118	125	123	98	97	89	80
Count > 0[5] (#):	52	23	54	62	61	36	45	29	30	65	7	93	25	98	63	67	45	72	36	57	26	45
POB >50%[4] (%):	85	0	100	100	100	7	89	1	0	100	0	100	0	100	57	94	0	98	1	97	0	89

Net value added histograms and statistics (ex. indexed)

		Percentiles[6] (%)	
Avg.[3,6]:	0.53%	100th:	78.75%
Std. error[6]:	0.18%	90th:	8.29%
Std. dev.[6]:	8.53%	75th:	3.40%
Skewness[6]:	2.35	50th:	-0.02%
Count (#):	2172	25th:	-3.23%
Count[6] > 0 (#):	1084	10th:	-6.99%
POB >50%[4,6]:	46.58%	0th:	-39.60%



Footnotes

1. Holdings from pre-2017 are inflation adjusted to 2017.
2. Average annualized benchmark return is the compound (geometric) average return of the annual fund-averaged benchmark returns as-reported to CEM Benchmarking.
3. Average annualized gross/net value added is the equal fund-weighted gross/net value added by year (i.e., Avg. defined in footnote 5) averaged with equal weight across years. Equal weighting years (as opposed to giving equal weight to all observations) is appropriate because statistically significant, non-zero, gross/net value added is observed in several years which demonstrates that observations from different years are not drawn from the same independent identical distributed random variables.
4. Odds actual proportion outperforming benchmark > 50% (POB > 50%) is the probability that the actual (rather than observed sample) proportion of portfolios outperforming the benchmark is greater than 50%. It combines the observed POB with the number of observations. A POB >50% above ninety-five (95) percent indicates there exists a high probability that some combination of profitable skill/strategy/bias led to the observed number of portfolios outperforming their benchmark.
5. Average (Avg.) net value added is the equal fund-weighted net value added by year. Standard error (std. error) is the one-sigma standard error on the fund average average net value added, equal to the standard deviation (std. dev.) divided by the square root of the number of observation (count). Skewness (skew.) is the Pearson's moment coefficient of skewness. The count greater than zero (count > 0) is the number of observations with net value added greater than zero.
6. Data and statistics amalgamated across years have been re-weighted by the inverse of the number of observations in a year. This is done to remove sampling biases causes by variations in the number of portfolios per year appearing in the dataset. Re-weighting does not change any conclusion we have drawn from our analysis.

4.2 Large cap. U.S. equity portfolios

Both the average value added and the proportion outperforming benchmark are reflective of an efficient market. The average gross value added of -0.02 +/- 0.13 percent is consistent with zero, and the proportion outperforming benchmark of 48.6% is consistent with outperformance / underperformance being no better than a coin flip.

However, once we include costs, the data reveal that large cap. U.S. equity portfolios underperform on average. The average net value added of -0.37 +/- 0.13 percent is three standard error different from zero. Active large cap. U.S. equities portfolios of U.S. large institutional investors lost value on average solely due to fees. The proportion outperforming benchmark, after costs, was 45.8% (only 569 of 1,241 portfolios produced positive net value added). This is materially below 50%: the odds that the actual (not just sample) proportion of portfolios outperforming the benchmark was over 50%, has dropped to 0.2% Looked at the other way, the odds that the actual proportion outperforming / underperforming is *worse* than a coin flip is 99.8%.

Value added by year shows several interesting features, despite their being zero gross value added over the whole sample period 1996-2017. First, it is clear that for several years, large cap. U.S. equity portfolios reliably outperformed their indices net of costs. For example, the odds of the actual proportion outperforming being over 50% was 100 percent (i.e., almost certain) in 2005, 2009 and 2013. On the other hand, in 1997, 1998, 2006, 2008, 2011, 2014, and 2016 the odds of the proportion outperforming being over 50% was zero. The pattern of outperformance / underperformance by year is portfolio-dependent. Some portfolios exhibit clear tilts and strategies relative to their large cap. U.S. equity benchmark that will be explored by us in future work.

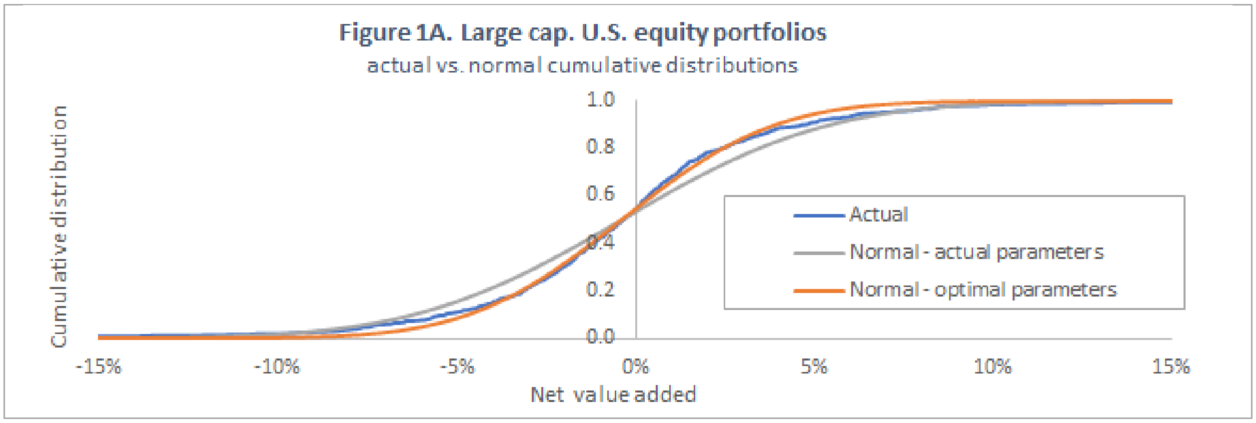
Of the statistics shown under 'Net value added by year', net value added is not correlated with standard error, standard deviation (i.e., funds taking more/less idiosyncratic risk), or skewness of the distribution of net value added. It is unsurprisingly correlated with the odds of the proportion outperforming being over 50% (ρ=77 percent).

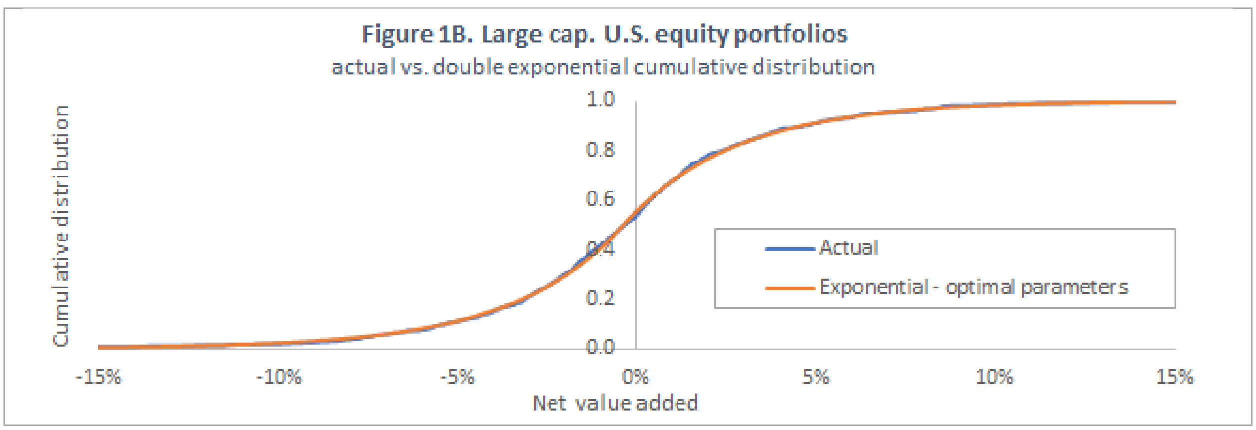
The distribution of net value added for large cap. U.S. equity is not a normal distribution (i.e., Gaussian distribution). To demonstrate, we show in Figure 1A the cumulative distribution of the actual data, the cumulative distribution of a normal distribution with the observed average and standard deviation (-0.37 percent and 4.54 percent respectively), and the cumulative distribution of a normal distribution with an optimal[3] average and standard deviation (-0.37 percent and 3.35 percent respectively). Clearly, neither fit the data well[4], with the actual distribution showing classic "fat tails".

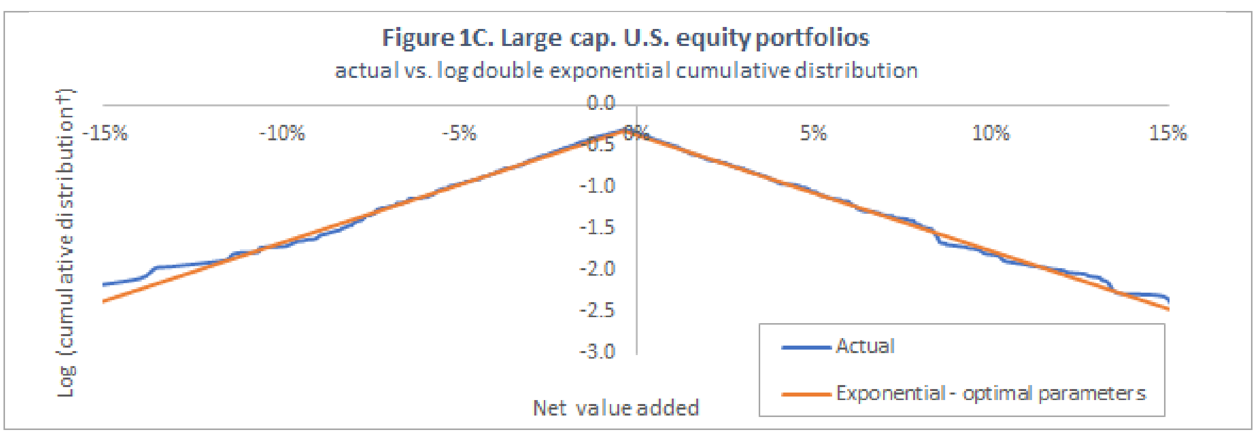
[3] Optimal values are determined by least squares.

[4] We have tried several other distributions such as Cauchy (Lorentzian), Student-t etc., and none provide a better result.

Figure 1A,B,C. Large cap. U.S. equity portfolio distributions. (A) Cumulative distribution of the actual data (blue) vs. the cumulative distribution of a normal distribution using (i) the observed average and standard deviation (grey), and (ii) optimal average net value added and standard deviation (orange). The misfit is typical of "fat-tail" distributions. (B) Cumulative distribution of the actual data (blue) vs. an exponential distribution with optimal parameters (orange). (C) Is the same as (B) except the cumulative distribution is plotted in semi-log form for net asset value less than average, and one minus the cumulative distribution for net asset value greater than average. If the actual distribution is exponential, the cumulative distribution plotted in this way should exhibit straight lines with an absolute slope equal to the standard deviation as shown.







† In Figure 1C, for net value added greater than the average (i.e., > -0.37%), log of one minus the cumulative distribution is shown in order to demonstrate the symmetry of the double exponential distribution around the mean for large cap. U.S. equity portfolios.

Rather, net value added for large cap. U.S. equity portfolios is described extremely well by a double exponential distribution[5] (i.e., a Laplace distribution) as shown in Figures 1B and 1C (in linear and semi-log form respectively). Indeed, the standard deviation of the observations (4.54 percent) and standard deviation to the optimal-fit exponential distribution (4.37 percent) are in very close agreement. The difference between the observations and best fit curve does show some structure (not shown), but it is very small, averaging less than 0.5 percent in magnitude. The structure is likely due to the finite size sample of our data which results in discreet jumps in the observed cumulative distribution.

We note that double exponential distributions are used widely in finance, from options pricing, credit risk modelling and log-pricing of equity markets [see for example Ref. 7,8,9], and so our finding use of it here is not novel. What appears to be new is the finding that a double exponential probability distribution describes well net value added (i.e., alpha) from actively managed large cap. U.S. equity portfolios.

How are we to interpret the result? Double exponential probability distributions describe a very specific type of random motion, namely Laplace motion. For our situation where the variable is value added, the Laplace motion is one of discreet jumps in value added randomly distributed in time. (This contrasts with a 'random walk' which has discreet and random jumps occurring at regular intervals.) Put simply, large cap. U.S. equity portfolios tend to track the market such that value added remains constant until an event occurs (e.g., a heavily weighted long/short position that substantially outperforms the broad market) that causes a material deviation from the index resulting in a jump in value added.

4.3 Small cap. U.S. equity portfolios

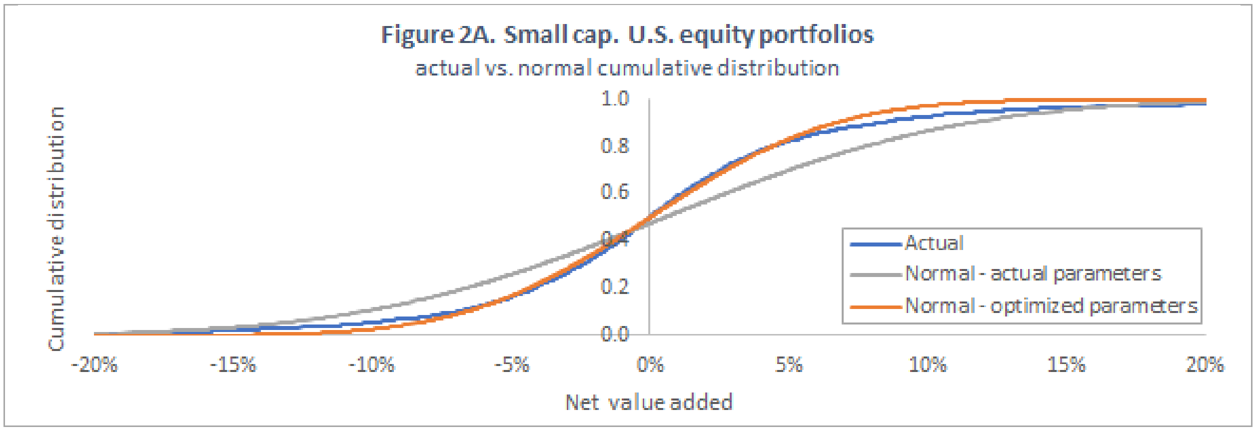
Both the high average gross value added and the high odds of the proportion outperforming being over 50% for small cap. U.S. equity portfolios are reflective of a market which is highly *inefficient* gross of costs. The average gross value added from small cap. U.S. equity portfolios over the whole sample period is +1.20 +/- 0.18 percent, larger than zero by more than six standard errors, making it extremely unlikely to be a product of chance alone. This together with the fact that 55.9 percent of portfolios outperformed their benchmarks is a clear demonstration of an inefficient market where actively managed portfolios on average beat the market before costs.

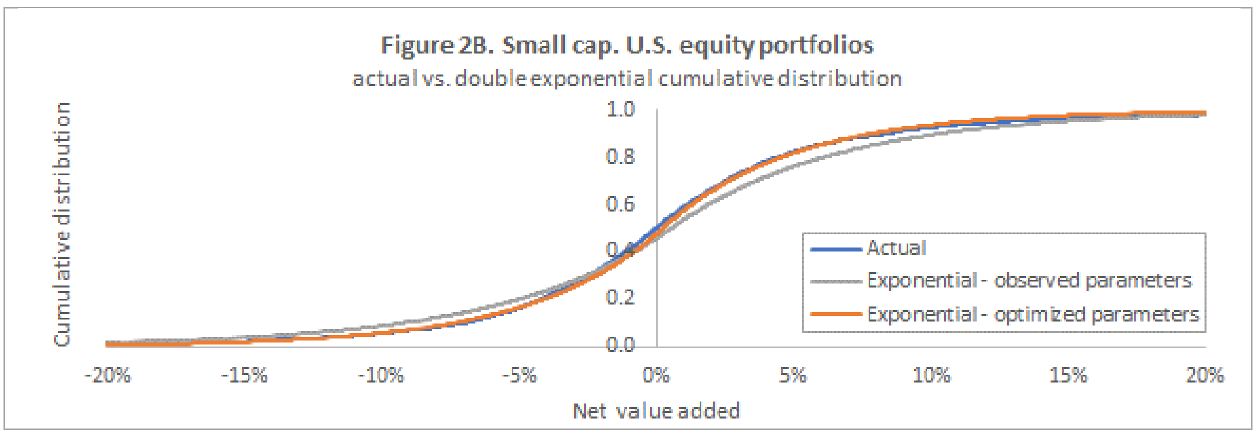
Once costs are included, the situation becomes more complicated. On the one hand, the average net value added over the whole sample period of +0.53 +/- 0.18 percent is almost three standard errors different from zero, suggesting only a 0.1 percent chance that the market is efficient net of costs. On the other hand, almost exactly one half of portfolios outperformed net of costs (49.90 percent; 1,084 of 2,172, only two different from an exact 50/50 split) which indicates that outperformance was a coin's flips chance. If outperformance and underperformance are equally likely, why is the net value added larger than zero?

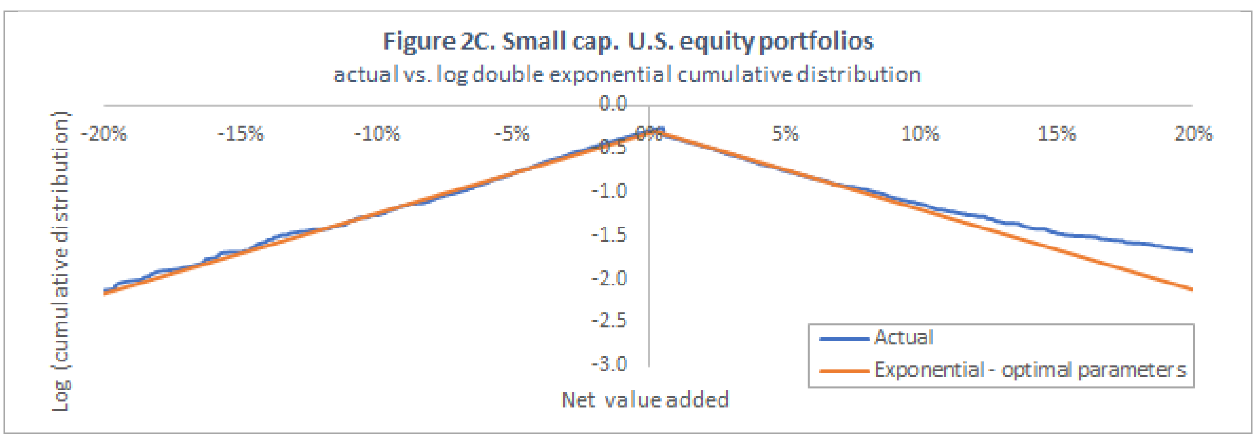
The reason small cap. U.S. equity portfolios see positive average net value added, yet only a coin's flip chance of outperforming, is that the distribution of net value added (and gross value added as well) is asymmetric, skewing positive over the whole sample period. The Pearson measure of skewness, as but one measure, is 2.35 for small cap. U.S. equity portfolios compared to only -0.18 for large cap. U.S. equity portfolios.

[5] A double exponential (or Laplace) distribution is given by $P(x) = \left(1/\sqrt{2\sigma^2}\right)\exp\left(-\frac{\sqrt{2}|x-\mu|}{\sigma}\right)$ where μ is the average and σ is the standard deviation.

Figure 2A,B,C. Small cap. U.S. equity portfolio distributions. (A) Cumulative distribution of the actual data (blue) vs. the cumulative distribution of a normal distribution using (i) the observed average and standard deviation (grey), and (ii) optimal average net value added and standard deviation (orange). The misfit is typical of "fat-tail" distributions. (B) Cumulative distribution of the actual data (blue) vs. an exponential distribution with optimal parameters (orange). (C) Is the same as (B) except the cumulative distribution is plotted in semi-log form for net asset value less than average, and one minus the cumulative distribution for net asset value greater than average. If the actual distribution is exponential, the cumulative distribution plotted in this way should exhibit straight lines with an absolute slope equal to the standard deviation as shown.







† In Figure 2C, for net value added greater than the average (i.e., > +0.53%), log of one minus the cumulative distribution is shown in order to demonstrate the the lack of symmetry of the double exponential distribution around the mean for small cap. U.S. equity portfolios.

Interestingly, a positively skewed net value added distribution is not associated with the distribution of results within any one year. Indeed, the skewness by year (as shown in Exhibit 2 under 'Net value added by year') for the most part bounces around zero, with a maximum of +1.07 in 1999 (an "up" year for active portfolios) and a minimum in of -1.60 in 2015 (a "neutral" year for active portfolios). Averaging the skewness across years yields 0.12; skewness is not a feature of annual net value added.

Instead, the skewness in net value added over the whole sample period (shown in Exhibit 2 under 'Net value added histograms and statistics') is produced by several years of highly positive net value added, more years than one would assume by chance alone. For example, during the formation of the "tech-bubble" from 1998-2000, three consecutive years saw positive value added that was between 3-4 standard errors different from zero. The formation and collapse of markets leading into the global financial crisis also saw three years of highly positive value added, 2005, 2007 and 2009.

The observed distribution of net value added over the whole sample period is unsurprisingly not a normal distribution, since the actual data is highly skewed and normal distributions have zero skewness. The lack of normality is shown in Figure 2A. For the same reason the distribution of value added is not normal, it is not a double exponential either, as was the case with large cap. U.S. equity portfolios. This fact is shown

first in Figure 2B where we have plotted the cumulative distribution of the observed data together with plots of the cumulative distribution for a double exponential distribution with: (i) the observed average net value added and standard deviation, and (ii) the optimal average net value added and standard deviation.

A more dramatic and much more interesting demonstration that the observed distribution of value added is not a double exponential is shown in Figure 2C where we have plotted in semi-log form the cumulative distribution for net value added for values less than the average, and one minus the cumulative distribution for values greater than the average. If the actual distribution was exponential, the data would form two straight lines of equal and opposite slope on either side of the average.

For net value added less than the average the double exponential distribution works well. For net value added greater than the average the double exponential does not work well past value equal to about 6 percent net value added. Thus, the source of deviations from the double exponential (and thus the cause of the skewness) are anomalously large and positive net value added that require explanation.

The primary cause of the anomalously large, positive value added is most likely contributions from a few select years, namely 1999 in the run-up to the tech bubble, and the post global financial crisis rebound of 2009. Clearly though not all portfolios pursued the same strategies, as the standard deviation in both years are among the largest, and the several portfolios in both years show several strong and negative net value adds. A detailed analysis and attribution of sources of value added to specific strategies and tilts is beyond the scope of this work and, we hope, will be presented in future research.

5 Discussion

A belief in efficient markets is a core motivation for many indexed investors. If it is not possible to outperform an index on average, or if outperformance is only a random probabilistic event, why try?

For large cap. U.S. equity portfolios, our data at CEM Benchmarking, sourced from and verified by the largest U.S. institutional investors, would seem to indicate that indexing is the most sensible option.

Efficiency of large cap. U.S. equity portfolios is somewhat unsurprising given the amount of research, analysis, and competition within the largest equity market in the world.

Small cap. U.S. equity portfolios on the other hand show clear evidence of market in-efficiency (as do several other asset classes that we at CEM Benchmarking have not publicly disclosed). The average gross value added is significantly greater than zero (+1.20 +/- 0.18 percent) and remains positive even after the inclusion of costs (+0.53 +/- 0.18 percent). However, the probability of a portfolio producing positive net value added in small cap. U.S. equities is no better than 50/50, in seeming contradiction to the positive average net value added.

The resolution of this apparent contradiction is found in the positive skewness of the net value added distribution. Within small cap. U.S. equity portfolios, positive net value added observations tend to be larger than negative net value added observations. Thus, the experience of the largest U.S. investors in small cap. equities is analogous to a coin flip where the benefits of winning outweighs the harm of losing.

Finally, the distribution of value added explored only briefly here deserve much more study. For large cap. U.S. equity portfolios, the distribution of net value added is almost perfectly described by a double exponential distribution. This tells us that value added for individual portfolios is governed by random Laplace motion where for periods value added is relatively constant, only to be disturbed by large positive/negative movements that occur at random intervals. For small cap. U.S. equity portfolios, we observe the same behavior except for an additional, highly positive, contribution to net value added associated with (so far publicly) undisclosed strategic investment decisions.

6 Appendix A

6.1 The data subset used in this research

Five important aspects of the data require description to appreciate and fully understand the results we present. Specifically, we have limited the research to;

1. two well defined asset classes, large and small cap. U.S. equity portfolios that are actively managed. (Similar or deeper research can be performed on other asset classes),
2. the years spanning 1996-2017[6],
3. funds located in the U.S. for the purpose of eliminating currency effects which make data consistency checks (see point 5 below) more complex and results less precise,
4. portfolios with a minimum of 33 percent active management[7],

[6] Data sets prior to 1996 are not sufficiently populated to warrant contribution to this research. Inclusion of them would give too much weight to data with too few observations (see the discussion of observation weighted average versus inverse year weighted average in section 3.2 'Methods'). Data post 2017 are not yet available.

[7] CEM receives performance, holdings and cost data at the asset class / implementation style (i.e., for public markets, internally/externally managed and indexed/actively managed) level. Benchmark performance data is obtained at the asset class level. To extract the active value added from blended portfolios (e.g., 50 percent internal indexed + 50 percent external active) we assume that gross value added from indexed portfolios are zero. For example, if a 50 percent active / 50 percent indexed portfolio returns 50 basis points of gross value added, we assume the active component returned 100 basis points of gross value added. In order to limit the potential noise from primarily indexed portfolios (value added from active management for a 95 percent indexed portfolio would receive a 20x multiplier), only portfolios with a minimum 33 percent active management were used.

5. portfolios with a benchmark return that strays no more than two standard deviations from the median benchmark return.

Extensive checks of the minimum allowance of active management (point 4) and the maximum allowance of benchmark return stray (point 5) were conducted concerning the impact on our conclusions on these choices. Our conclusions are static for an allowance of active management ranging from about 25 percent or so up to 75 percent, and for benchmarks within 4 standard deviation of the median.

Regarding point 4, if the minimum allowance of active management were set too low, portfolios with low amounts of active management (typically large funds) began dominating the results due to large multipliers applied to value added. Conversely, if the minimum allowance of active management is set too high, smaller funds are over-represented in the results.

Regarding point 5, if the maximum allowance of stray from the median benchmark return was set too low, only portfolios with their performance measured against the most common benchmarks are included. If the maximum allowance of stray from the median benchmark was set to high, data with questionable benchmarks might adversely affect the results.

6.2 Sources of bias and their removal

6.2.1 Year-to-year variability in participation: Equal-year weighting

One potential source of bias in our results is the year-to-year variability in participation in CEM Benchmarking services. As shown in Exhibits 1 and 2 and discussed in Section 4, net value added by year is measurably different from zero for several of the years in the sample period, meaning that observations in different years do not come from the same underlying statistical distribution. If in taking averages we equal weighted each observation, the equal weighting would give out-sized influence to those years in which funds were more likely to participate in CEM Benchmarking services.

In order to make our results insensitive to the number of funds participating in CEM Benchmarking services, we weight all observations proportional to the inverse of the number of observations in that given year. In terms of the average (gross/net) value added over the whole sample period 1996-2017, this is equivalent to equal weighting each year's (equal fund-weighted) annual average[8]. This method of analysis – inverse-year weighting – is extended to all performance measures including standard deviations and standard errors, measures of skewness, number of observations greater than zero, and percentile rankings.

6.2.2 Variability in sample size: Odds the proportion outperforming is over 50%

The 'odds of the proportion outperforming being over 50%' is derived from binomial statistics. As used here, it measures the probability that the actual (rather than observed sample) proportion of portfolios of this type outperforming the benchmarking is greater than 50%.

The importance of using binomial statistics can be illustrated with a simple example. If 4 of 10 portfolios outperformed their benchmark, it is not improbable that the portfolios had some intrinsic skill and had a

[8] Our primary conclusions are not sensitive to the way in which averages are taken. For example, for large cap. U.S. equity portfolios, net value added over the whole sample period is either -0.20 +/- 0.12 percent (equal weighted) or -0.37 +/- 0.13 percent (inverse-year weighted). For small cap. U.S. equity portfolios, net value added over the whole sample period is either +0.61 +/- 0.18 percent (equal weighted) or +0.53 +/- 0.18 percent (inverse-year weighted).

50/50 chance or better of outperformance but were simply unlucky (the probability that they had a 50/50 chance or better is actually 38 percent). By contrast, if 450 of 1000 portfolios outperformed their benchmarks, it is almost certain (the probability is effectively zero) that portfolios did not have a 50/50 chance (of better) of outperforming as luck would be much more unlikely of an explanation of the low frequency of observed outperformance even though the fraction of portfolios outperforming is higher than in the prior scenario.

The odds of the proportion outperforming being over 50% thus eliminates a second potential source of bias; for different years or for different asset classes, the number of observations is not equal, and binomial statistics tells us how to account for the difference in sample size.

In terms of statistical significance, if the odds of the proportion outperforming being over 50% is in excess of 95 percent, we can say with 95 percent confidence that portfolios had a 50/50 chance or better at beating their benchmark, due to investing skill or strategy or some profitable bias against the benchmark. Conversely, if the odds of the proportion outperforming being over 50% is below 5 percent, we can say with 95 percent confidence that portfolios had a 50/50 chance or worse of beating their benchmark.

7 About CEM Benchmarking

CEM Benchmarking is a Toronto based provider of investment cost and performance benchmarking for large institutional investors including pension funds (defined benefit and defined contribution), sovereign wealth funds, buffer funds, and others. For information on benchmarking with CEM or other data inquiries please contact:

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What gets measured gets managed